A hardCORE model for constraining an exoplanet's core size

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The Model

Understanding the internal structure of an exoplanet is a crucial step in determining its habitability. Unfortunately, mass and radius alone cannot reveal how much iron, silicon or water a solid planet is made of, let alone its core radius fraction (CRF). Our model exploits two boundary conditions in order to solve for both the minimum core radius fraction (CRF) and the maximum core radius fraction (CRF). We note that this model assumes that the planet is fully differentiated, that the core is not made of any element denser than iron (e.g. no uranium cores), and that if there is an outer envelope, it has negligible mass.

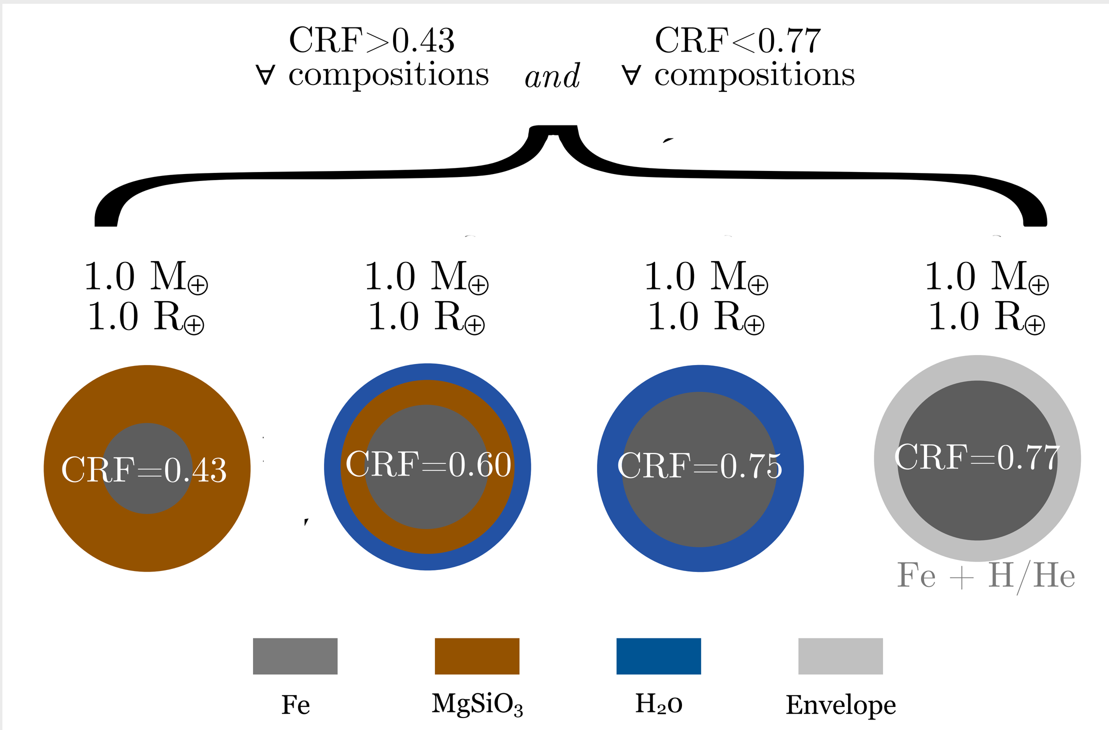

Fig. 1. Four example planets with very different interiors that all share the same mass and radius and thus are indistinguishable with current observations. All satisfy having a CRF exceeding 43%, the value of the two-layer iron-silicate model (first sphere). The largest iron core size allowed is depicted by the lowest sphere, where the volatile envelope contributes negligible mass.

Method

In order to calculate CRF_{min}, we parametrically interpolate the theoretical two-layer iron-silicate estimates of mass and radius from Zeng & Sasselov (2013). Our model, which we dub `hardCORE`, can be easily inverted to provide a unique solution for CRF_{min}. By retraining and cross-validating our model, we find that the mean error of our model is 0.045% and the maximum error is 0.24%. Determining CRF_{max} is far more straight-forward. We simply take the 100% iron mass-radius models, and directly compute the expected radius of a pure iron planet given an observed mass, $R_{iron}(M_{obs})$. The maximum core radius fraction is then easily computed as

$$CRF_{max} = R_{iron}(M_{obs})/R_{obs}.$$

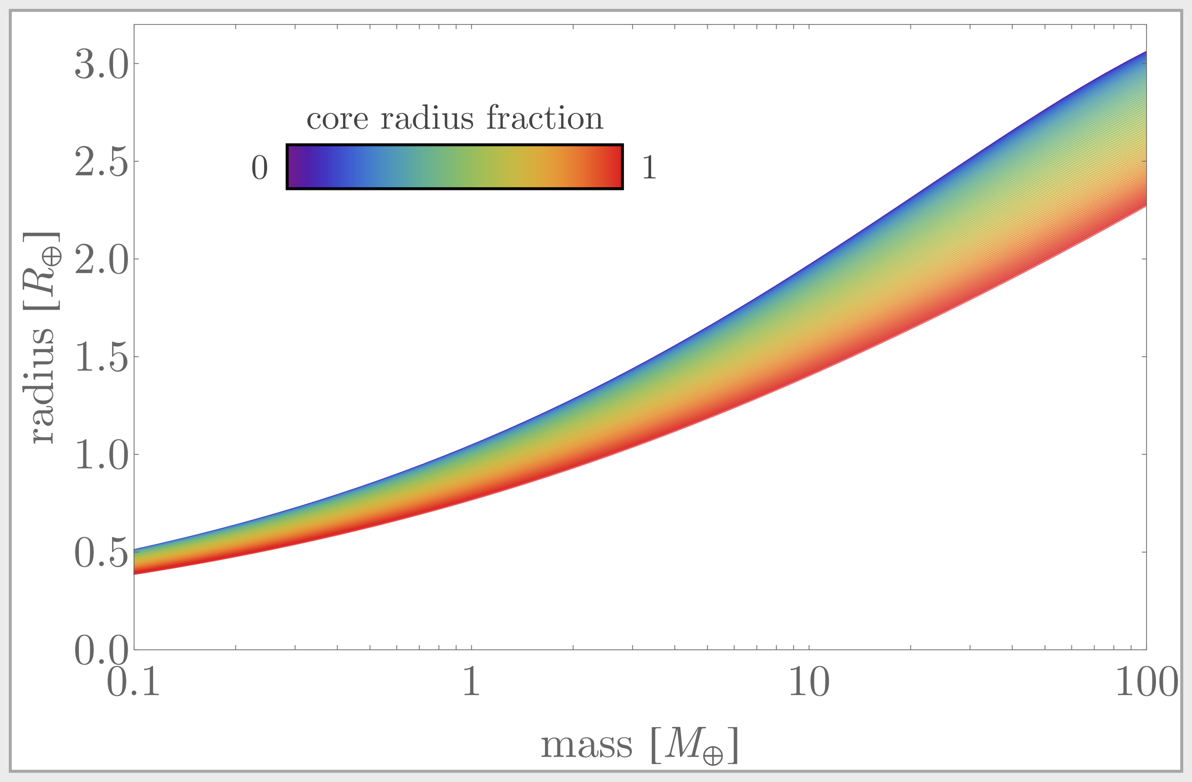

Fig. 2. Interpolated theoretical mass-radius relations for a silicate-iron two-layer solid planet for various core radius fractions (CRFs), based off Zeng & Sasselov (2013). All interpolations for CRFs between 0 and 1 are seventh-order polynomials.

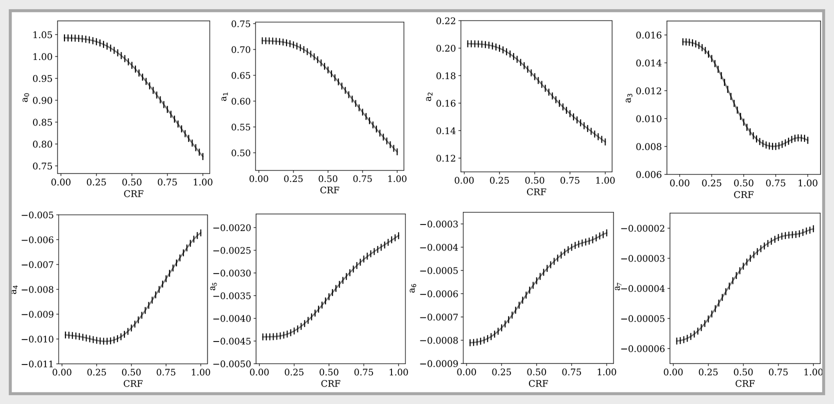

Fig. 3 We are then motivated to describe the dependence of the polynomials with respect to the CRF, by making the coefficients polynomial functions themselves to create a parametrized interpolation.

Applications

With both the minimum and maximum CRFs acting as a bounding box, a marginal CRF can also be inferred by sampling in between. As an example application, we implement `hardCORE` on the rocky planet Kepler-36b. Using its real mass-radius joint posterior distribution consisting of 10^4 samples, our model yields a CRF_{min} and CRF_{max} posterior from which we can draw random samples to create a CRF_{marg} posterior. Our CRF_{marg} for Kepler-36b = 0.64±0.10. For comparison, the CRF_{marg} for a synthetic Earth yields 0.60. This in general agrees with previous conclusions of Kepler-36b: the planet appears to be compatible with having an Earth-like interior.

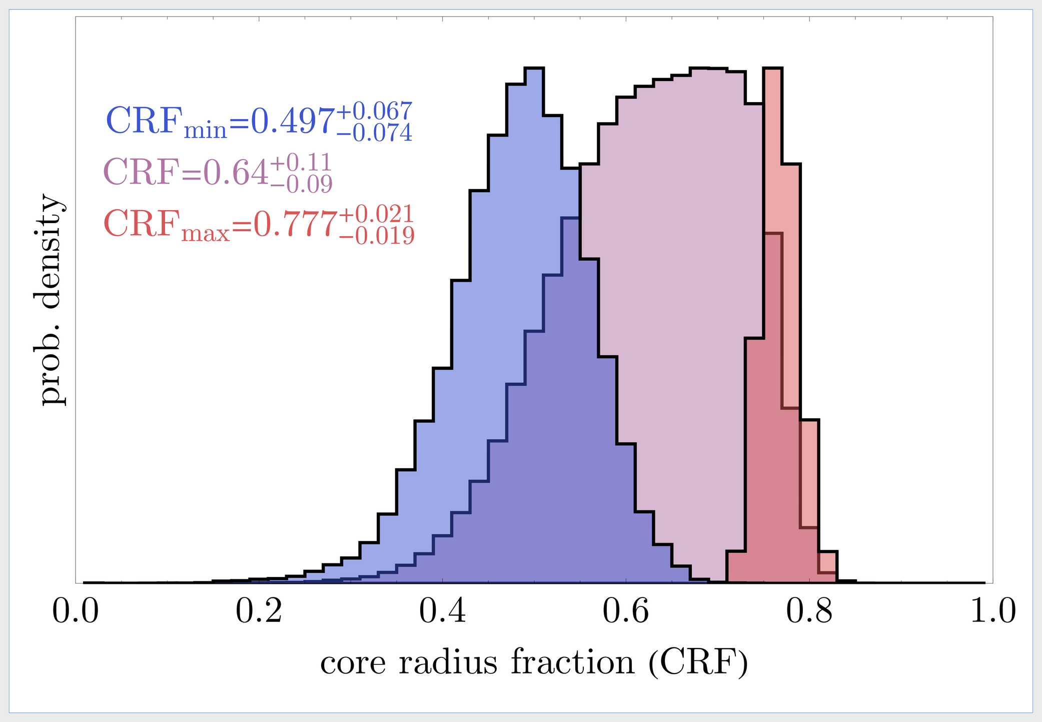

Fig. 5. Posterior distribution of the minimum CRF (left), maximum CRF (right) and marginalized CRF (center) for Kepler-36b, based off the joint mass-radius posterior from Carter et al. (2012) and the model presented in this work. Posterior heights normalized to be equivalent.

Results

A basic and important question to ask is what kind of precisions on a planet's mass and radius lead to meaningful constraints on CRF_{min}, CRF_{marg}, and CRF_{max}? In other words, what is the correspondence we might expect between {(ΔM/M), (ΔR/R)} and (ΔCRF/CRF)? We investigate what kind of precisions on a planet's mass and radius lead to meaningful constraints on CRF_{marg}? We conduct a sensitivity analysis for our model and find that the radius is the dominant constraint for the CRF. CRF_{marg} appears to saturate to ~10%. This implies that no better than 10% precision can ever be obtained on the CRF using just mass and radius alone, providing a clear goal post for observers interested in compositions.

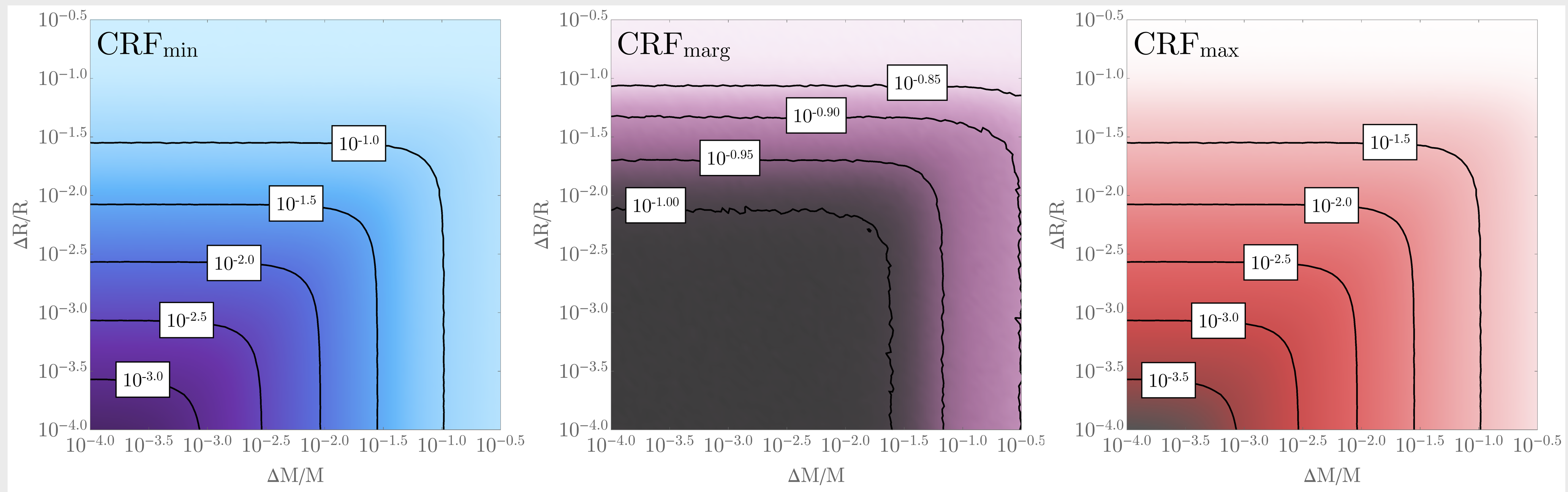

Fig. 5. Contour plots of sensitivity analysis of CRFmin, CRFmarg and CRFmax. For example, ito obtain a precision of 10% on CRFmin, we require a measurement on the mass better than 11% and a measurement on the radius better than 3%.

References

Zeng, L. & Sasselov, D. D. 2013. PASP 125, 227

Carter, J. A., Agol, E., Chaplin, W. J., et al. 2012, Science, 337, 556

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Public code: github.com/gsuissa/hardCORE

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